Gomal Univ. J. Res.
Vol. 10 No.1 pp 23-26 (1990)

Printed in Pakistan

ON THE EULER'S SUMMATION FORMULA

Mohammad Abbas, Abdul Latif
Mathematics Department, Gomal University, D.I.Khan.

Received 07-08-89
Accepted 30-09-89

ABSTRACT

In this paper we give simpler form of the Euler's summation formula and extension of this result is also given.

INTRODUCTION

The well known generalization of the Riemann integral is the Riemann-Stieltjes integral, which can be defined several accepted ways [see 1, 2, 3,]. One of its definition is given in [2] as follows: let f and α be two real-valued functions defined and bounded on [a,b]. For a partition $P=\left\{x_0, x_1, x_2, \quad , x_n\right\}$ of [a,b], $\Delta\alpha_i = \alpha(x_i - 1)$. Asum of the form $S(P,f,\alpha)=\sum_{a=1}^{n} f\ (t_i)\ \Delta\alpha_i$, $t_i \in \left[x_{i-1}, x_i\right]$ is called a Riemann-Stieltjes sum of f with respect to α. We say f is Riemann-Stieltjes integrable with respect to α on [a,b]. If there exists a number J with the property that, "For every ε 70, there exists a partition $P\varepsilon$ of [a,b] such that corresponding to each partition P finer than $P\varepsilon$, $|S(P,f,\alpha) - J| < \varepsilon$. Such a number J is uniquely determined and is called Riemann-Stieltjes integral, denoted by $\int_a^b f\ (x)\ d\alpha\ (x)$.

The following result (Euler's summation formula) is proved in [2,P201].

Theorem A. If f has a continuous derivative f on [a,b], then

(i) $\sum \quad f(n) = \int_a^b f(x)\,dx + \int_a^b f(x)\ (x-[x])\,dx + f(a)\ (a-[a])\ -$