# COMPARATIVE STUDY OF SOLAR FLARE NORTH-SOUTH HEMISPHERIC AND K-INDEX GEOMAGNETIC ACTIVITY TIME SERIES DATA USING FRACTAL DIMENSION

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## ABSTRACT
*The solar flare activity is associated with the sunspots activity. The solar flare index data (monthly flare index cycles "20, 21, 22, 23", available on world data center) is used starting from 1966 to 2008 in this research work. The main purpose of this research is to compare the solar flares activity data of north hemisphere, south hemisphere and total hemisphere with K-index activity data using fractal dimension. Persistency (1 < D < 1.5) is observed for each solar cycle of each hemisphere along with the total hemisphere and compare with the associated K-index activity data. Persistency analysis of solar flares activity and K-index activity data will be useful to understand the effects of solar flares dynamics on the K-index dynamics. There is strong bond found between solar flares north hemisphere and K-index data. Hurst exponent is also obtained with the help of fractal dimension (H = 2-D) to understand the long rang dependence (self-similarity) of solar flares and K-index activity dynamics. The results represent that each solar flare cycle (north, south and total hemisphere) and K-index activity data along with total time series data of both are regular dynamics and continue to long term in future. The complex of time series can be analyzed using Fractal dimension (local property) of the data while smoothness of time series be observe by Hurst Exponent (universal property).*

**Keywords:** *Solar Flare, K-index, Fractal Dimension, Geomagnetic, Hurst Exponent*

## INTRODUCTION
The sudden brightness that is an explosive event on the sun is defined as solar flares. Solar flares, coronal mass ejection and high-speed solar wind streams are often causes the damaging disturbances within the magnetosphere of earth that is in the atmosphere and on the surface (Mirmomeni & Lucas, 2009). Solar flares are explosive on the sun and one of the forms of solar activity which is more powerful according to the images of the sun (Jilani & Zai, 2013). Many studies are categorized in the solar-terrestrial field solar flares activity of sun as the most important one of the solar events that is affecting the earth (Ataç & Özgüç, 2001). The coronal mass ejection (CMEs) and the solar flare (SF) are the major types of energy that is continuously released by the sun (Kilcik, 2009; Yousafzai, 2003). The energy which is released by solar flares as energetic particles is equal $10^{32}$ erg (Weiss,

2006; Thomas, 2008). There are three classes of solar flare activity normally according to their brightness; they are X-class, M-class and C-class.

## LITERATURE REVIEW

The major storms on the earth can be produced by the X-class solar flares, at polar latitudes the M-class solar flares can cause radio blackouts and there is no significant consequences of C-class solar flares (Li, Liang, Yun & Gu, 2002; Thomas, 2008). There is a strong relation between solar flares and sunspots, as solar flares occur inside or near to sunspots but not necessarily (Rabello, 2012). During each 11-year cycle of sunspots the polarity reverses the sunspots embedded in the northern hemisphere from a magnetic south polarity. Due to this local polarity reversal the global polarity is reversal (Pesnell, 2012). The sun's dipole field will change with the perception that the north pole magnetic field of sun will control from geographic north pole to geographic south pole. The reversal of polarity occurs during the minimum sunspots and because of this reversal the sun has 22-year magnetic cycle (Thomas, 2008).

The variations in space brings the changing on the magnetosphere and environment of earth, the solar activity variations can be observed on the earth in terms of geomagnetic activity variations (Hanslmeier, 1999). The solar activity variation is recorded on the earth in terms of geomagnetic activity indices. Mayaud defined the geomagnetic activity indices that are furnished since1868 to nowadays (Pesnell, 2012). With the help of Km (Am) and Kp (Ap) indices the level of geomagnetic activity can be observed. It would be immensely valuable to understand the effects on Kp-indices by the various solar activity indices in solar physics point of view (Hanslmeier, 1999). The fractal dimension is define as under

$$fractal\ dimension = \frac{no\ of\ small\ pieces}{magnification\ level} \tag{1.1}$$

Through fractal dimension or Hurst exponent we can observed the persistency (regular and predictable) of any time series data (Hassan, 2014).

$$D + H = 2 \tag{1.2}$$

So with the help of above relation we can analyze the local as well as universal property of solar flares and K-index activity data (Thomas, 2008).

## MATERIAL AND METHODS

The solar flares cycles 20, 21, 22 and 23 of North, South and total hemisphere are used in the research work. The K-index geomagnetic activity time series data are distributed according to the solar flares cycles.

**Solar Flare & K-index Geomagnetic Activity (Fractal Dimension)**

The disturbances on the magnetosphere and surface of earth occur due to the solar activity indices. The space weather is depends on different solar activity indices that causes the substorms and geomagnetic storms and they are more effective for the climate of the earth (Rathore, 2011; Pesnell, 2012). The one of the main task for space mission is to predict the chaotic behavior of long term solar activity indices (Hanslmeier, 1999). A variation of 27-day is depending on the rotation period of the solar activities and conditioned by the occurrence of magnetic structures and group of large sunspots (Pries, 1982). Thus variations of global indices over different time scales of solar activity indices and hence their time series can be explained and defined as time fractal curves. To understand the regular and irregular behavior of the dynamics of the system, fractal methods can be used (Salakhutdinova, 1998).

The dimension that is used most widely is box dimension or box-counting (Falconer, 2007). Because of the relative ease of empirical estimation and mathematical calculation this method is most popular. In 1930s the definition of fractal dimension has been different termed entropy dimension, information dimension, capacity dimension, Kolmogorov entropy, logarithmic density and metric dimension. To avoid confusion, we always refer box-counting dimension (Falconer, 2007). The fractal dimension analysis contains more insight into the complex behavior and dynamical structure of system (Michael F. Barnsley, 1988). Fractal dimensions can be used to compare different numbers which are related to the fractals (Hassan, Abbas, Ansari & Jan, 2014). The Fractal dimension is one of the important technique that can be defined a connection of real world data which can be estimated by experiments approximately. The fractal dimension relation to Hurst exponent is defined in the following table.

| H | FD | Correlation | Nature of Process |
|---|---|---|---|
| >0.5 | <1.5 | Positive | Persistent |
| =0.5 | =1.5 | Zero | Brownian |
| <0.5 | >1.5 | Negative | Anti-Persistent |

The above relationship is summarized by (Sugihara & May, 1990) which represent nature of time series data as well as correlation (Falconer, 2007; Hassan, 2014). For the parameter (H) the Hurst exponent introduced by the scale that is ranging from 0 to 1. It is the indication of regularity or irregularity (chaos) behavior of the time function in the term of their persistent and anti-persistent respectively. When value of Hurst exponent parameter (H) increases to 1 then the time function will more quasi-regular (persistence). Similarly the time function will be irregular (anti-persistence) for the value of (H) close to 0. The

Hurst exponent is first elaborated by Mandelbrot (1983) that is related to the fractal dimension (D) by $D = 2 - H$ (Salakhutdinova, 1998; Pesnell, 2012).

## RESULTS AND DISCUSSIONS

Fractal dimension of each solar flares cycle (20, 21, 22 and 23) are showing persistency (D < 1.5) depicts table 1and Fig.1. It represents that all the observed solar flares cycles are quasi-regular and they are predictable depicts in table 1. The K-index cycles (associated to solar flares cycles) are also persistent (D < 1.5) and they are quasi-regular in nature table 1 depicts and Fig.2. So it is also predictable in future. Our results indicate that the fractal dimension of north hemisphere solar flares cycles are approximately same to the K-index cycles depicts in table 1 along with the total time series data. Second significant condition is found for total hemisphere solar flares cycles including the total time series data. The fractal dimension of each K-index cycle is slightly less than the fractal dimension of corresponding each solar flares cycle (north hemisphere) except 23$^{rd}$ cycle depicts in table: 1 (see Fig.2).The 24$^{th}$ solar flares cycle is in progress. After the solar flares cycle 23 the data is not available on world data center. With the help of available K-index activity data on world data center after the 23$^{rd}$ cycle, the result shows that the fractal dimension of K-index activity cycle from 2009 to 2014 is found to be 1.388.

The K-index activity is representing persistent after 23$^{rd}$ cycle. By the relation of fractal dimension of solar flare cycle and the corresponding K-index activity data we can predict that the 24$^{th}$ solar flare will be Quasi-regular (persistent) and its fractal dimension will be close to fractal dimension of the K-index data particularly for the north hemisphere. Our results show that the relationship between the north hemispheric solar cycles and the K-index activity data, which is distributed according to the solar flare cycle is strongest than the south hemisphere and the total hemisphere cycles of solar flares in terms of fractal dimension. According to the results obtained, the solar flares cycle 20 of north, south and total hemispheres are the roughest (less quasi-regular) among the all. For K-index activity cyclic data cycle 23 is found roughest (less regular dynamics) and cycle 21 is the least rough (more regular dynamics) among the all (see Fig.1). Results are depicts in table 2, which show that each solar flares (north, south and total hemisphere) and K-index activity data are self-similar (long-rang dependence).

The correlation of fractal dimensions of solar flares cycles of north, south and total hemispheres with K-index activity data are respectively 0.42605, -0.11999 and 0.352751. Results indicate that the K-index activity cyclic data is strongly associated with the north solar flares cycles as compare to the south hemisphere and total hemispheres solar flares data. So our instrument which is established between solar flare cycle (north, south and total hemisphere) and K-index activity cyclic data indicates the strongest relation for north solar flares cycle with the K-index activity data as compare to the south hemisphere. For geomagnetic indices (K-index) data collection, more observatories located in north hemisphere as compare to the south hemisphere, it may be a reason for strongest relationship between solar flare activity north hemisphere and K-index activity data.

## CONCLUSION

In solar physics point of view the shifting of solar flare data with respect to the north hemisphere is more persistent (quasi-regular and predictable) and smooth as compare to the south hemisphere solar flares data. Fractal dimension (D) represents the roughness (local property) of the time series data of solar flares and K-index activity while Hurst exponent (H) indicates the smoothness (universal property) of the data. Our results confirm that the solar flares (north, south and total hemispheres) cycles and K-index activity are following the quasi-regular dynamics which are also self-similarity. The strong relationship of K-index and north hemisphere solar flares cycle data indicates that the solar flares dynamics and geomagnetic (K-index) variations continue to the long term. Results show that the total hemisphere (global) solar flares activity data and K-index data have the second strong relationship. The relation between south solar flares activity and K-index is found to be inverse proportion. By the discussion we conclude that the climatic variability with respect to the north hemisphere is more predictable than the south hemisphere. The study will be useful to understand and predict the climatic change on the earth's hemispheres as well as for the space weather.

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## Annexures 1 Tables

Table: 1 Comparison of Fractal Dimension of Solar Flares North, South and Total Hemispheric Cycles with K-Index Activity Cycles

| Cycle | Duration | Fractal Dimension (North) | Fractal Dimension (South) | Fractal Dimension (Total) | Fractal Dimension (K-index) |
|---|---|---|---|---|---|
| 20 | 1966.01-1975.04 | 1.271 | 1.463 | 1.394 | 1.125 |
| 21 | 1975.05-1986.06 | 1.072 | 1.282 | 1.047 | 1.062 |
| 22 | 1986.07-1996.1 | 1.121 | 1.154 | 1.048 | 1.11 |
| 23 | 1996.11-2008.12 | 1.122 | 1.176 | 1.112 | 1.156 |
| 20-23 | 1966.01-2008.12 | 1.134 | 1.241 | 1.15 | 1.144 |

Table: 2 Comparison of Hurst Exponent of Solar Flares North, South and Total Hemispheric Cycles with K-Index Activity Cycles)

| Cycle | Duration | Hurst Exponent (North) | Hurst Exponent (South) | Hurst Exponent (Total) | Hurst Exponent (K-index) |
|---|---|---|---|---|---|
| 20 | 1966.01-1975.04 | 0.729 | 0.537 | 0.606 | 0.875 |
| 21 | 1975.05-1986.06 | 0.928 | 0.718 | 0.953 | 0.938 |
| 22 | 1986.07-1996.1 | 0.879 | 0.846 | 0.952 | 0.89 |
| 23 | 1996.11-2008.12 | 0.878 | 0.824 | 0.888 | 0.844 |
| 20-23 | 1966.01-2008.12 | 0.866 | 0.759 | 0.85 | 0.856 |

## Annexures 2 Figures

Figure 1 Comparison of Solar Flares Cycles (20-23) and K-Index Acitivity (1966-2008)

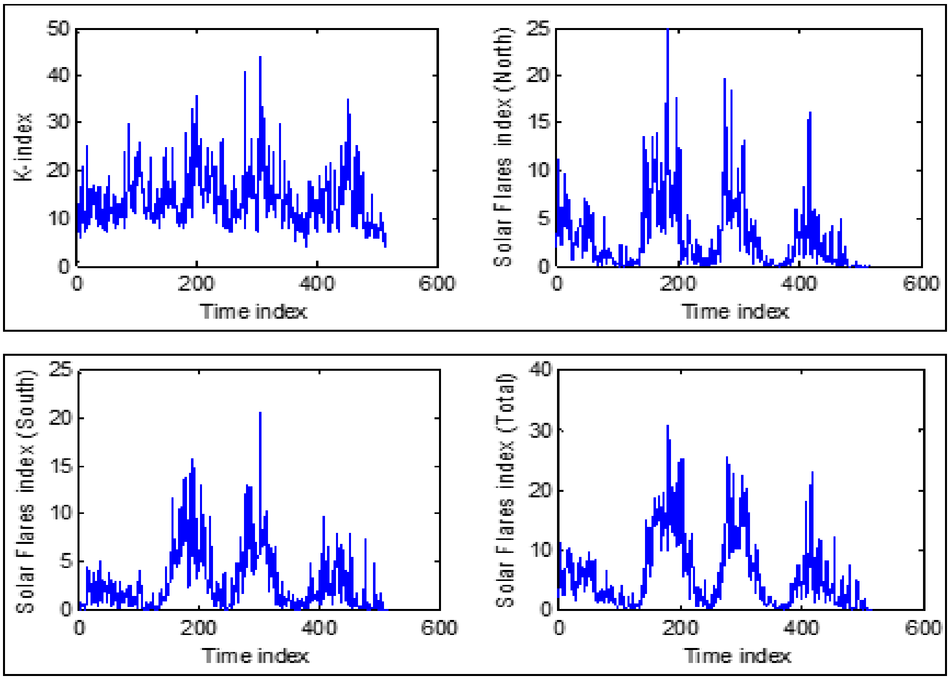


Figure 2 Comparison of Fractal Dimensions of Solar Flares Activity (North, South and Total Hemisphere)