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IRREGULAR S_3 -ALGEBRAS

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ABSTRACT

We characterize S_3-algebra containing three, four and five elements by left-maps and show that there is an irregular such algebra of order 3, four irregular, non-isomorphic such algebras of order 4 and seventy irregular, non-isomorphic such algebras of order 5 respectively. Moreover we investigate that out of these algebras of order 3,4 and 5; one, three and seventeen are weakly positive implicative respectively.

In 1966, Y. Imai and K. Iseki introduced the concept of BCK-algebras ([1]). In the same year K. Iseki in [2] generalized that concept and obtained the notion of a BCI-algebra as follows:

A BCI-algebra is a non-empty set X with a binary operation * and a constant 0 satisfying the following axioms:

For all x, y, z $\in$ X,

BCI 1 $(x*y)*(x*z) \le z*y$,

BCI 2 $x*(x*y) \le y$,

BCI 3 $x \le x$,

BCI 4 $x \le y,\ y \le x$ imply $x = y$,

BCI 5 $x \le 0$ implies $x = 0$.

Where $x \le y$ if and only if $x * y = 0$

If in BCI 5"$x \le 0$ implies $x = 0$" is replaced by "$0 \le x,\ \forall x \in X$" then X is a BCK- algebra.

The following are true in a BCI-algebra:

(1) $(x*y)*z = (x*z)*y$,

(2) $x*0 = x$,

(3) $x \le y$ implies $x*z \le y*z$ and $z*y \le z*x$,

(4) $(x*z)*(y*z) \le x*y$ ([3]).

Let X be a BCI-algebra and B = { $x \in X$: $0 \le x$ } be its BCK- part, then X is a proper BCI-algebra if X-B $\ne \phi$ ([3]).

It is well known that every BCK-algebra is a BCI-algebra but not vice versa.

Further in [3] K. Iseki has proved the following proposition:

Proposition 1. In a BCI-algebra X if $b \in B$, $x \in X\text{-}B$, then $x*b,\ b*x \in X\text{-}B$.

A non-empty subset A of a BCI-algebra X is called a sub-algebra of X if a, b $\in$ A imply $a*b \in A$ ([4]).

Let X be a BCI-algebra and B its BCK-part, then X is called a p-semisimple algebra if B = {0} ([5]). In [9], S. K. Goel, have shown that;