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SOME NEW EXACT SOLUTIONS OF EQUATIONS GOVERNING THE STEADY MOTION OF AN INCOMPRESSIBLE SECOND GRADE FLUID

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ABSTRACT

Equations governing the steady motion of an incompressible second grade fluid are transformed into a new single PDE. Some new exact solutions are determined and eight applications of some of the exact solutions are indicated.

INTRODUCTION

Recently Naeem [1] , by introducing stream function Ψ, the complex variables z. $\bar{z}$ and the complex functions, transformed the system of partial differential equations governing the steady motion of an incompressible second grade fluid into a single partial differential equation in five unknown complex functions. Naeem then described a procedure for finding the solutions of the single complex partial differential equations. The solutions obtained are new and admit, as particular cases, the solutions obtained by Siddique et al [2].
This paper is an extension of Naeem's work. The plan of this paper is as follows: In section (2); we give the equations of motion and transform them into a new single partial differential equation in seven complex functions which reduce to PDE in [1] by assuming two of the complex functions zero. We then determine new exact solutions. In section (3), we discuss the applicability of the solutions to physically possible situations. In section (4) we give the conclusions on the present work.
This paper admits, all the solutions in [1] , as particular cases, by appropriately choosing the arbitrary complex functions or the arbitrary constants in some of the solutions there in.

1. EQUATIONS OF MOTION

The basic equations governing the steady plane flow of an incompressible second grade fluid, in the absence of external forces, are [1,3]

$$U_x + V_y = 0 \tag{2.1}$$

$$\begin{aligned} \rho(U U_x + V U_y) + p_x = \mu \nabla^2 U + \alpha_1 \{ & [2 U U_{xx} + 2 V U_{xy} + 4 U_x^2 + 2 V_x (U_y + V_x)]_x \\ & + [U (V_x + U_y)_x + V (V_x + U_y)_y + 2 U_x U_y + 2 V_x V_y]_y \} \\ & + \alpha_2 \{ [4 U_x^2 + (V_x + U_y)^2]_x \} \end{aligned} \tag{2.2}$$

$$\begin{aligned} \rho(U V_x + V V_y) + p_y = \mu \nabla^2 V + \alpha_1 \{ & [U (V_x + U_y)_x + V (V_x + U_y)_y + 2 U_x V_y + 2 V_x V_y]_x \\ & + [2 U V_{xy} + 2 V V_{yy} + 4 V_y^2 + 2 U_y (V_x + U_y)]_y \} \\ & + \alpha_2 \{ [4 V_y^2 + (U_y + V_x)^2]_y \} \end{aligned} \tag{2.3}$$

where U, V are the components of the velocity, p is the pressure, ρ is the constant fluid density, μ is the coefficient of viscosity, and α_1 and α_2 are the normal stress moduli.

The system of equations (2.1- 2.3) on introducing the stream function Ψ, transforms to the following system of partial differential equations