

## A COMPUTATIONAL ANALYSIS OF COMPLEMENTARY ERROR FUNCTION USING POWER SERIES AND CONTINUED FRACTION METHODS

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### ABSTRACT

A successful numerical study of the Complementary Error Function, denoted as  $erfc$ , using Power Series and Continued Fraction methods, has been made. For small value of  $\beta$ , the Power Series method is used while for large value of  $\beta$ , the Continued Fraction method is used. Both methods are found robust. The error analysis for these methods is also studied.

### INTRODUCTION

The objective of this study is to make the computational analysis of the Complementary Error Function [1, p.163]

$$erfc(\xi) = \frac{2}{\sqrt{\pi}} \int_{\xi}^{\infty} e^{-y^2} dy \quad (1)$$

where  $\xi = \beta/2$  and  $0 < \beta < \infty$ .

But, 
$$erfc(\xi) = 1 - erf(\xi) \quad (2)$$

Where,

$$erf(\xi) = \frac{2}{\sqrt{\pi}} \int_0^{\xi} e^{-y^2} dy, \quad -\infty < \xi < \infty$$

To evaluate the above integral, power series and continued fraction expansions are used for  $\beta$  small and  $\beta$  large, respectively, with an interval (1.5, 2.5) of overlap for  $\beta < 2$  (shown in Table. 1) and  $\beta \geq 2$  (shown in Table. 2).

### COMPUTATIONAL METHODS

Power Series Expansion:

The error function in terms of the power series expansion [2, p.93] is given by

$$erf(\xi) = \frac{2}{\sqrt{\pi}} \int_0^{\xi} e^{-y^2} dy, \quad -\infty < \xi < \infty \quad (3)$$