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A THREE LEVL PSEUDO-SPECTRAL METHOD FOR VORTICITY EQUATIONS

Abdur Rashid and Ghulam Mustafa
Department of Mathematics, Gomal University, D.I.Khan. Pakistan.
Department of Mathematics, Islamia University, Bahawalpur. Pakistan.

Abstract:

This paper develops a Three-Level Fourier Pseudo Spectral method for solving two-dimensional vorticity equations. It proves the generalized stability of the scheme, which gives convergence estimations depend on the smoothness of the solution of the vorticity equations.

1. Introduction:

Let $\xi(x_1, x_2, t)$ and $\psi(x_1, x_2, t)$ be the vorticity function and the stream function respectively. ν is positive constant, $f_j(x_1, x_2, t)$, $(j = 1, 2)$ and $\xi_0(x_1, x_2, t)$ are given.

Let $\Omega = \{ (x_1, x_2) \mid -\pi \le x_1, x_2 \le \pi \}$,

Consider the following tow-dimensional vorticity Equations

$$\begin{cases} \dfrac{\partial \xi}{\partial t} + \dfrac{\partial \psi}{\partial x_2}\dfrac{\partial \xi}{\partial x_1} - \dfrac{\partial \psi}{\partial x_1}\dfrac{\partial \xi}{\partial x_2} - \nu\left(\dfrac{\partial^2 \xi}{\partial x_1^2} + \dfrac{\partial^2 \xi}{\partial x_2^2}\right) = f_1, \quad in \quad \Omega \times (0,T] \\ -\dfrac{\partial^2 \psi}{\partial x_1^2} - \dfrac{\partial^2 \psi}{\partial x_2^2} = \xi + f_2, \quad in \quad \Omega \times (0,T] \\ \xi(x_1, x_2, 0) = \xi(x_1, x_2), \quad in\ \Omega. \end{cases} \tag{1.1}$$

There are many papers concerning the Finite Difference method, Spectral and Pseudo Spectral methods for solving (1.1) see [1-5]. In these papers the stability and convergence of two-level scheme are proved, by using the first order time discretization. The overall convergence rates are $O(\tau + N^{-s})$, which is not satisfactory result for the sharp contrast between the first order temporal discretization and the infinite order of spatial Spectral or Pseudo Spectral spatial discretization.

To improve the accuracy level in temporal discretization, the work of [6-8] is applied to construct Three-Level Pseudo Spectral scheme for solving two-dimensional vorticity equation. The stability and convergence of the approximate scheme were thoroughly analyzed, which resulted in a