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A NOTE ON THE COMMUTATIVITY AND EQUALITY OF TWO PROJECTIONS IN THE CALKIN ALGEBRA

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ABSTRACT

In [3], it has been shown that if P and Q are any two projections in B(H) such that PQP=QPQ, then P and Q commute. The aim of this note is to show that the same proof yields when the underlying algebra is considered to be the Calkin algebra $K(=B(H)/K(H))$. Futhermore, an attempt is made to find the condition under which two projections could be equal.

INTRODUCTION

Let H be a Hilbert space and let B(H) be the algebra of all bounded linear operators on H. A projection in B(H) is defined as an operator P which is self-adjoint (i.e. $P^* = P$) and idempotent (i.e. $P^2 = P$). We say that two projections P and Q in B(H) are orthogonal if $PQ = 0$, otherwise non-orthogonal. In [3], it has been shown that for any two projections P and Q in B(H), the commutativity relation $PQ = QP$ is equivalent to $PQP = QPQ$. In [2], G.A. Khan and G. Rehman gave much shorter proof of Rehder's result [3] and also showed that for any two projections P and Q in B(H), the equality relation $P = Q$ is equivalent to $PQP = QPQ$ whenever P and Q are non-orthnogonal.

Let H be a complex separable infinite dimensional Hilbert space and let B(H) be the algebra of all bounded linear operators on H. We denote by K(H) the two-sided ideal of compact operators on H. Let $T \varepsilon B(H)$ and [T] be the image of T in the Calkin algebra $K(=B(H)/K(H))$. Obviousely K is the algebra of equivalent classes