

ON THE SPECTRUM OF AN ELEMENTARY TYPE OPERATOR

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ABSTRACT

Let H be a complex separable Hilbert space and let $B(H)$ be the algebra of all bounded linear operators on H . Let $\{A_1, \dots, A_n\}$ and $\{B_1, \dots, B_n\}$ be two commuting families of self-adjoint operators in $B(H)$. In this paper we are concerned with the investigation of the spectrum of the elementary type operator $\Gamma: B(H) \rightarrow B(H)$ defined by $\Gamma(X) = \sum_{i=1}^n A_i X B_i$ for all X in $B(H)$.

More precisely, we show that

$$\text{Sp}(\Gamma; B(B(H))) = \left\{ \sum_{i=1}^n \lambda_i \mu_i : \lambda_i \in \text{Sp}(A_i; B(H)), \mu_i \in \text{Sp}(B_i; B(H)) \right\}.$$

INTRODUCTION

Let X be a Banach space, and let $B(X)$ be the algebra of all bounded linear operators on X . If $T \in B(X)$, then the spectrum of T denoted by $\text{Sp}(T; B(X))$ is defined to be the set

$$\text{Sp}(T; B(X)) = \{ \lambda \in \mathbb{C} : T - \lambda I \text{ is not invertible in } B(X) \}.$$

For A, B in $B(X)$, the operator $R : B(X) \rightarrow B(X)$ is defined by