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S_4-ALGEBRAS OF ORDER THREE, FOUR AND FIVE BY SELF-MAPS

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Abstract.

The BCI-algebras have been Classified, namely S_i-algebras, i= 1,2,3,4([8]). We characterize S_4-algebra of order 3,4 and 5 by self maps and show that there is one fully non-associative such algebra of order three, one associative and one neutral such algebra of order 4 and only one non-isomorphic fully non- associative such algebra of order 5 respectively.

Preliminaries.

A BCI-algebra is an algebra (X,*, 0) of the type (2, 0) with the following conditions:-

For any $x, y, z \in X$,

(1) $(x * y) * (x * z) \le z * y$,

(2) $x * (x * y) \le y$,

(3) $x \le x$,

(4) $x \le y, y \le x$ imply $x = y$,

(5) $x \le 0$ implies $x = 0$.

Where $x \le y$ if and only if $x * y = 0$ ([1]).

If in (5), "$x \le 0$ implies $x = 0$" is replaced by "$0 \le x$ for all $x \in X$" then X is called a BCK-algebra.

Let X be a BCI-algebra and $M = \{ x \in X : 0 \le x \}$ be its BCK-part. Then X is called proper BCI-algebra if $X-M \neq \emptyset$ ([1]).

Note that every BCK-algebra X is a BCI- algebra but the converse is not true.

A non-empty subset A of a BCI-algebra X is called a subalgebra of X if $a, b \in A$ implies $a * b \in A$ ([12]).

W.A. Dedek ([3]) defined that a BCI-algebra satisfying

(6) $(x * y) * (z * u) = (x * z) * (y * u)$

is a medial BCI-algebra.