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IRREGULAR S_3-ALGEBRAS

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ABSTRACT

We characterize S_3-algebra containing three, four and five elements by left-maps and show that there is an irregular such algebra of order 3, four irregular, non-isomorphic such algebras of order 4 and seventy irregular, non-isomorphic such algebras of order 5 respectively. Moreover we investigate that out of these algebras of order 3,4 and 5; one, three and seventeen are weakly positive implicative respectively.

In 1966, Y. Imai and K. Iseki introduced the concept of BCK-algebras ([1]). In the same year K. Iseki in [2] generalized that concept and obtained the notion of a BCI-algebra as follows:

A BCI-algebra is a non-empty set X with a binary operation * and a constant 0 satisfying the following axioms:

For all $x, y, z \in X$,

BCI 1 $(x * y) * (x * z) \leq z * y$,

BCI 2 $x * (x * y) \leq y$,

BCI 3 $x \leq x$,

BCI 4 $x \leq y$, $y \leq x$ imply $x = y$,

BCI 5 $x \leq 0$ implies $x = 0$.

Where $x \leq y$ if and only if $x * y = 0$

If in BCI 5"$x \leq 0$ implies $x = 0$" is replaced by "$0 \leq x$, $\forall x \in X$" then X is a BCK- algebra.

The following are true in a BCI-algebra:

(1) $(x * y) * z = (x * z) * y$,

(2) $x * 0 = x$,

(3) $x \leq y$ implies $x * z \leq y * z$ and $z * y \leq z * x$,

(4) $(x * z) * (y * z) \leq x * y$ ([3]).

Let X be a BCI-algebra and $B = \{ x \in X: 0 \leq x \}$ be its BCK- part, then X is a proper BCI-algebra if $X - B \neq \phi$ ([3]).

It is well known that every BCK-algebra is a BCI-algebra but not vice versa.

Further in [3] K. Iseki has proved the following proposition:

Proposition 1. In a BCI-algebra X if $b \in B$, $x \in X-B$, then $x * b$, $b * x \in X-B$.

A non-empty subset A of a BCI-algebra X is called a sub-algebra of X if $a, b \in A$ imply $a * b \in A$ ([4]).

Let X be a BCI-algebra and B its BCK-part, then X is called a p-semisimple algebra if $B = \{0\}$ ([5]). In [9], S. K. Goel, have shown that;