

A COMPUTATIONAL ANALYSIS OF COMPLEMENTARY ERROR FUNCTION USING POWER SERIES AND CONTINUED FRACTION METHODS

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ABSTRACT

A successful numerical study of the Complementary Error Function, denoted as $erfc$, using Power Series and Continued Fraction methods, has been made. For small value of β , the Power Series method is used while for large value of β , the Continued Fraction method is used. Both methods are found robust. The error analysis for these methods is also studies.

INTRODUCTION

The objective of this study is to make the computational analysis of the Complementary Error Function [1, p.163]

$$erfc(\xi) = \frac{2}{\sqrt{\pi}} \int_{\xi}^{\infty} e^{-y^2} dy \quad (1)$$

where $\xi = \beta/2$ and $0 < \beta < \infty$.

But,
$$erfc(\xi) = 1 - erf(\xi) \quad (2)$$

Where,

$$erf(\xi) = \frac{2}{\sqrt{\pi}} \int_0^{\xi} e^{-y^2} dy, \quad -\infty < \xi < \infty$$

To evaluate the above integral, power series and continued fraction expansions is used for β small and β large, respectively, with an interval (1.5, 2.5) of overlap for $\beta < 2$ (shown in Table. 1) and $\beta \geq 2$ (shown in Table. 2).

COMPUTATIONAL METHODS

Power Series Expansion:

The error function in terms of the power series expansion [2, p.93] is given by

$$erf(\xi) = \frac{2}{\sqrt{\pi}} \int_0^{\xi} e^{-y^2} dy, \quad -\infty < \xi < \infty \quad (3)$$