CHANGE IN THE LAYERS OF EARTH IN TERM OF FRACTIONAL DERIVATIVE: A STUDY

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KEYWORDS	ABSTRACT
Crust, Mantle, Outer Core, Inner Core, Density, Gravitational Acceleration, Pressure, Elasticity, Temperature, Fractional Derivative.	The Earth is composed of four layers mainly known as (crust, mantle, inner core and outer core). Change in these layers caused an earthquake. The layers of earth are changed with depth, depending upon many factors, which affects the structure of earth. Change in inner core is very large and continuous and depends upon many factors like, temperature, pressure, density, gravitational acceleration, viscosity and elasticity. Temperature plays an important role in earthquake factors, many researchers thought that this change in layers is a discrete, but literature review reveals that change in layers caused by different factors is continuous and minor. So, we deal this in fractional calculus, to find this continuous change. Modifying formula into fractional change in the pressure, temperature and density are calculated by selecting the relevant formula and are compared with discrete values which are more accurate. In future this can be used to calculate change in the layers continuously which will be helpful in predicting a major change.

INTRODUCTION

Earthquake is defined as vibration of Earth surface. Tectonic plates are moving inside the earth with very high speed. When some change appears in the layers of earth by changing its factors, a disturbance appears in tectonic plates and tectonic plates change their way in which it is moving. By this, tectonic plates collide and energy release in the form of seismic waves. By this process an earthquake occurs. An Earthquake is a natural phenomenon which is most devastating (Alfe, Gillan & Price, 2007). It is also affecting the environment and lifestyle of the people. Earthquake cause many changes in the environment which is also divided into two forms like the primary and secondary. Primary effects are caused by earthquake and the secondary effects are caused by primary. These effects are called the earthquake environment effects (Serva, Blumetti, Esposito, Okumura, Porfido, Reicherter & Vittori, 2015). For the assessment of earthquake, two scales are used one is magnitude and the other is the intensity scale (Baize & Scotti, 2016).

Intensity scale is used for the earthquake environment scale. There are three different intensity scales (ESI 2007, EMS-98 and MM) for the assessment of Earthquake. By the comparison of the results ESI 2007 provides accurate result for ground affects but it is used for less populated areas. In the most populated areas and manmade structure, EMS-98 and MM is preferable then ESI 2007 (Hayakaw, Rozhnoi & Solovieva, 2013). Konsuk and Aktas (2013) has shown statistical analysis of data of 231-earthquakes of magnitude five and more than five from July,12,1900 to October 23, 2011 in Turkey between Latitude and Longitude 39.00° - 42.00° N and 26.00° - 45.00° E. Two functions of magnitude were derived, probability density function and cumulative function. Hayakaw shows that in the generation of seismoionospheric perturbations, the relative significance of magnitude and earthquake depth at middle latitudes, was explored by utilizing EQs near propagation path during period of 3 years (2005-2007), from Japanese LF transmitter, JJY (Fukushima) to reception station in Russia at Petropavsk-Kamchatsky (PTK).

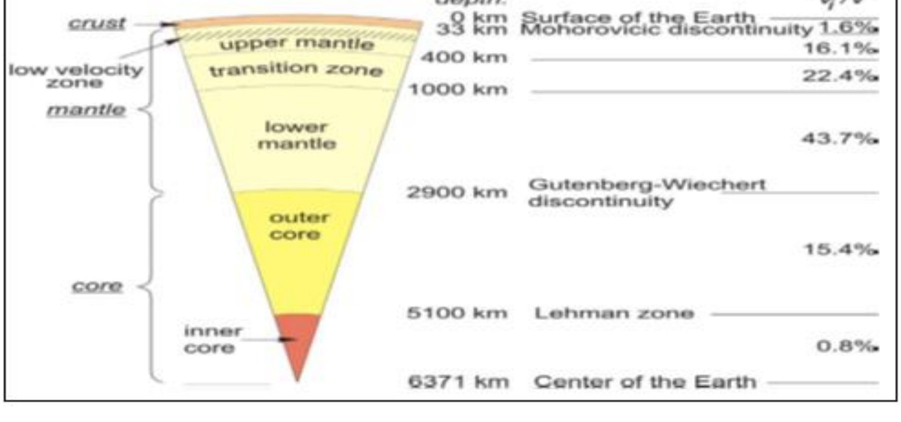
The result illustrated that at middle latitudes, when depth (100 Km) was compared to the magnitude in producing seismoionospheric perturbations, it was highly non-significant factor. The sea EQs result in Izu-Bonin and Kurile-Kamchatka arcs was in contrast with EQs result of Japan that were of fault-type. The coupling mechanism of the lithosphere-atmosphere-ionosphere was used to interpret these differences (Urata, Duma & Freund, 2018). The electrical currents are generated by solar winds in ionosphere. On the surface of the Earth, fluctuations in magnetic field were caused by these currents. In the presence of Earth magnetic field 'B', the electrical current induction and electromagnetic force generation were resulted by these fluctuations that penetrated the interior of the Earth, is also called as Lorentz force $F = J \times B$. A logarithmic measurement of deviation of magnetic field i.e. Kp index was determined to observe the relation between EQs and the Lorentz force that act at the beginning times of strong EQs.

The Kp index (time varying) gives the value of J, which determines F. The deviations of the Kp index were arranged by aligning both central times and the times of main shocks of earthquakes. Due to the lifting of only the geomagnetic effect as carving the relief, this arrangement method is very significant tool in the image processing. Though, the internal dynamics are more statistical. The coupling mechanism of the lithosphere-atmosphere-ionosphere was used to interpret these differences. For comparison of time sequence of the hypothesized random EQs to actual, a combination of conventional statistical method and anew devised method was used. The result was that different patterns of the Kp index mostly correlated to the beginning of EQs. The depending factors of this correlation were seismic areas and earthquake's magnitude. The stronger the earthquake lead to more close association of Kp surge.

LITERATURE REVIEW

The statistical importance of almost 100% is gained for Kp variability, bringing into line with more earthquakes in region of Pacific Rim. The earthquake occurrence is considered to be influenced by solar activities and in past century along with recent years, the relation between solar activities and the earthquakes were deeply studied (Gutenberg, 1959). Earth surface cover by 71% salt water sea and other 31% by seven continental land masses. Earth is inclined on his axis by 23.4°.Magnetic field of earth produced by inside the melt outer core zone of the planet. The third planet of the sun is earth and fifth greatest planet of the sun. The mass of earth is 5.972×10^{24} kg and diameter are 12756.3 km. Earth is divided in four layers (Crust, Mantle, inner & outer core). Earth has 21% oxygen, 78% nitrogen and 1% all other gases with traces of water CO_2 and argon. The diameter of earth is 8,000 miles. The scientists believe our earth and moon is formed from our solar system. Average distance of earth from sun is 93 million miles. Earth is radius 3,959 miles. Earth complete one rotation about sun is 23 hours, 56 minutes and 4 seconds (Choudhury et al., 2016).

Figure 1 Pictorial view of layers of Earth



Factors of Earthquake

There are some factors of the Earth due to which the Earthquake causes and leads to the higher intimidations to the human and material resources.

Density

Density is a ratio of its mass and volume. The density of the earth is measured by its mass and volume. The volume of the earth is measured by the artificial satellite and by newton laws of gravity the mass of the earth is calculated. Density is also defined as the degree of compactness of the object. The substance of the same volume with different mass gives the different density. Density of the earth varies in different layers of the earth. The value of density increases from top of earth to bottom of earth. The density of the earth is greater in the inside of the earth. The internal structure of the earth is detected by the density of the earth. The maximum value of density is 12500 kgm^{-3}.

$$\rho = \frac{\mathbf{Me}}{\mathbf{Ve}}$$

Pressure

Pressure is the force per unit area which is applied on the surface of body. Pressure is increase with depth. We can say that pressure is directly proportional to depth. The value of pressure is maximum at center of earth which is 3.64 ·1011 Nm^{-2}. We see the in graph the value of pressure at different depth. Pressure of the inner core is very high.

$$\mathrm{P} = \int_{\mathrm{R}}^{\mathrm{r}} \mathrm{g}\,\partial \mathrm{dr}$$

Gravitational Acceleration

Gravitational acceleration is the acceleration of a body cause by the gravity force. The gravity of the earth is different at different depth. The maximum value of gravity is 10.7 ms^{-2}. The value of gravity is 9.8 ms^{-2} at the surface of earth and minimum at the center of the earth which is zero. The value of gravity at different depth is given in the graph.

$$\mathrm{g_r} = \frac{\mathrm{KM_r}}{\mathrm{r^2}}$$

Temperature

According to the physic, thousands of years ago the earth is the part of the sun. an accident happened; the earth is separated from sun. The temperature of the earth is very high. We know that heat always moves hot body to cool body so the temperature of the earth decreasing. Temperature is measured by boreholes for few kilometers. Temperature increase with the depth.

$$q = \lambda \text{ grad } T$$

Fractional Calculus

Fractional Calculus is a new powerful tool which has been recently employed to model complex biological systems with non-linear behavior and long-term memory. In spite of its complicated mathematical background, fractional calculus came into being of some simple questions which were related to the derivation concept; such questions as while the first order derivative represents the slope of a function, what a half order derivative of a function reveal about it? Finding answers to such questions, scientists managed to open a new window of opportunity to mathematical and real world, which has arisen many new questions and intriguing results. For example, the fractional order derivative of a constant function, unlike the ordinary derivative, is not always zero.

Nature of Fractional Derivative

The a-th derivative of a function $f(\mathrm{x})$ at a point x is a local property only when 'a' is an integer; this is not the case for non-integer power derivatives. In other words, it is not correct to say that the fractional derivative at x of a function $f(x)$ depends only on values of f very near x, in the way that integer-power derivatives certainly do. Therefore, it is expected that the theory involves some sort of boundary conditions, involving information on the function further out. The fractional derivative of a function to order 'a' is often now defined by means of the Fourier or Mellin integral transforms.

Fractional Derivative of a Basic Power Function

Let us assume that $f(x)$ is a monomial of the form

$$f(x) = x$$

The first derivative is as usual

$$f'(x) = \frac{d}{dx} f(x) = kx^{k-1}$$

Repeating this gives the more general result that

$$\frac{d^a}{dx^a}(x^k) = \frac{k!}{(k-a)!} x^{k-a}$$

Which, after replacing the factorials with the Gamma Function gives us

$$\frac{d^a}{dx^a}(x^{-k}) = \frac{\Gamma(k+1)}{\Gamma(k-a+1)} x^{k-a}, k \geq 0$$

For negative integer power k, the gamma function is undefined and we have to use the following relation:

$$\frac{d^a}{dx^a}(x^{-k}) = (-1)^a \frac{\Gamma(k+a)}{\Gamma(k)} x^{-(k+a)}, k \geq 0$$

MATERIAL AND METHOD

Asia is the most seismically active region due to continuous change in layers. For this reason, fractional calculus is proposed to study the change in layers of earth by considering the factors. The earthquake catalogue has been analyzed for the studied region (Saqlain, Irshad, Bashir, Murtaza & Jafar, 2018) and the same catalogue is also used in this study. The factors are characterized by the specific parameters that are assumed to be continuous within the zone, these parameters are:

- ✓ Change in Temperature
- ✓ Change in Pressure
- ✓ Change in Density

Calculations

Due to change is layers, volume changes. Since volume depends on mass and density, also mass of each layer is different, with the passage of time in different depths layers' changes which results in the change of volume.

Change in Volume

So, change in volume in term of fraction is given by,

$$\frac{d}{dm}^{\alpha}(v) \qquad 0 < \alpha \leq 1$$

Change in Density

Due to the change in mass of layers the density changes since density depends on volume

$$\frac{d\partial}{dr} = \frac{-KM_r \partial_r}{r^2(v_p^2 - v_s^2)}$$

$$\frac{d\partial}{dr} = \frac{d\partial}{dr} = \frac{-KM_r \frac{d}{dm}^{\alpha}(v)}{r^2(v_p^2 - v_s^2)} \qquad 0 < \alpha \leq 1$$

Change in Temperature

Since the pressure and density changes the temperature is also changes

$$q = \lambda \text{ grad } T$$

$$q = \lambda \frac{d}{dp}^{\alpha} (gradT) \qquad 0 < \alpha \leq 1$$

Change in Pressure

Due to the change in density of layers the pressure changes since the mass of layers' change

$$P = \int_R^r g\partial dr$$

$$P = \int_R^r g \frac{d}{dm}^{\alpha} (v)\, dr \qquad 0 < \alpha \leq 1$$

Where,

$$\partial = \frac{d}{dm}^{\alpha} (v)$$

Calculation of Density

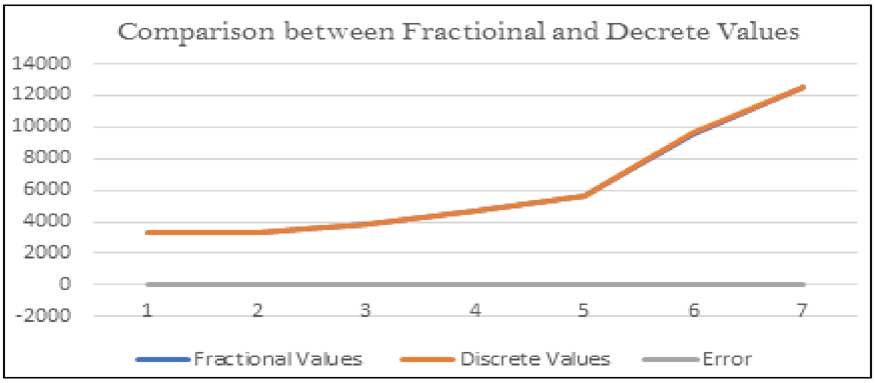
Now we calculate the values of density at different layers of Earths by varying depths, it can be given by substituting

$$\alpha = 1 \quad into \quad \frac{d\partial}{dr} = \frac{d\partial}{dr} = \frac{-KM_r \frac{d}{dm}^{\alpha} (v)}{r^2(v_p^2 - v_s^2)}$$ such that $0 < \alpha \leq 1$ For α =1

Table 1 Values of Density for α =1 and Varying Depths

Layer of Earth	Depth	Fractional Values	Discrete Values	Error
Crust	0	3300.01	3300	0.01
Mantle	33	3300.09	3300	0.09
	470	3879.1	3880	-0.9
	1000	4650.4	4650	0.4
	2900	5660.0004	5660	0.0004
Outer core	2900	9600.005	9700	0.005
Inner Core	6371	12500.04	12500	0.4

Figure 2: Comparison between Fractional and Discrete Values



Calculation of Temperature

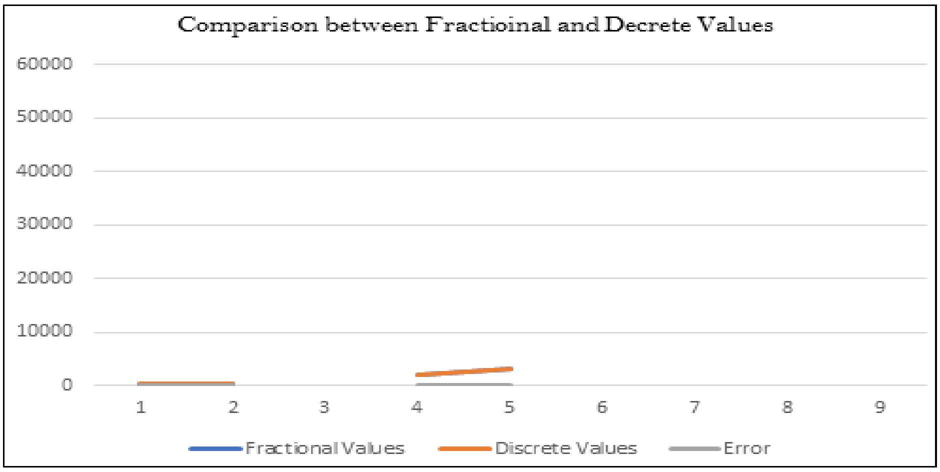
Now we calculate the values of density at different layers of Earths by varying depths, it can be given by substituting

$\alpha = 1 \quad into\ q = \lambda \frac{d}{dp}^{\alpha} (gradT)$ such that $0 < \alpha \leq 1$ For $\alpha = 1$

Table 2: Values of Temperature for α =1 and Varying Depths

Layer of earth	Depth	Fractional Values	Discrete Values	Error
Crust	0	303.15	303	0.15
Mantle	33	423.001	423	0.01
	470	1900.05	1900	0.5
	1000			
	2900	3000.03	3000	0.03
Outer core	2900			
	5100	50000.08	5000	0.8
Inner core	5100			
	6371	7000.004	7000	0.004

Figure 3 Comparison between Fractional and Discrete Values



Calculation of Pressure

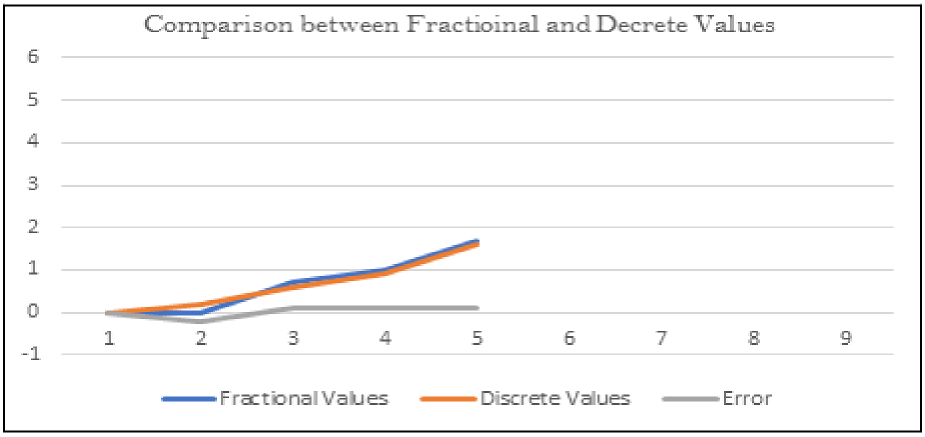
Now we calculate the values of density at different layers of Earths by varying depths, it can be given by substituting

$\alpha = 1 \quad into\ P = \int_R^r g \frac{d}{dm}^{\alpha} (v)\, dr$ such that $0 < \alpha \leq 1$ For $\alpha = 1$

Table 3: Values of Pressure for α =1 and Varying Depths

Layer of earth	Depth	Fractional Values	Discrete Values	Error
Crust	0	0.00001	0	0.00001
Mantle	33	0.003	0.2	-0.197
	470	0.7	0.6	0.1
	1000	1	0.9	0.1
	2900	1.7	1.6	0.1
Outer core	2900			
	5100	3.05	3.5	-0.45
Inner core	5100			
	6371	5.65	3.64	0.01

Figure 4 Comparison between Fractional and Discrete Values



Calculation of Density

Now we calculate the values of density at different layers of Earths by varying depths, it can be given by substituting

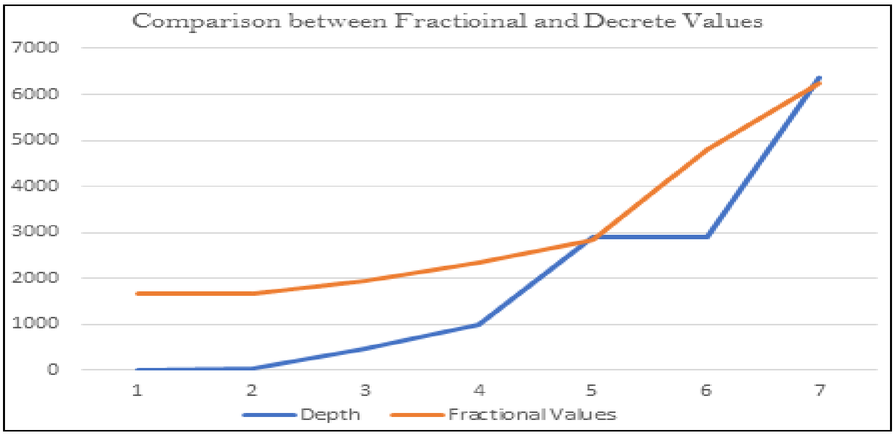
$$\alpha = \frac{1}{2} \quad into \quad \frac{d\partial}{dr} = \frac{d\partial}{dr} = \frac{-KM_r \dfrac{d}{dm}^{\alpha}(v)}{r^2(v_p^2 - v_s^2)} \qquad \text{such that } 0 < \alpha \leq 1$$

For $\alpha = \frac{1}{2}$

Table 4 Values of Density for $\alpha = \frac{1}{2}$ and Varying Depths

Layer of earth	Depth	Fractional Values
Crust	0	1650.004
Mantle	33	1650.048
	470	1939.60
	1000	2325.2
	2900	2830.002
Outer core	2900	4800.02
Inner Core	6371	6250.01

Figure 5 Comparison between Fractional and Discrete Values



Calculation of Temperature

Now we calculate the values of density at different layers of Earths by varying depths, it can be given by substituting

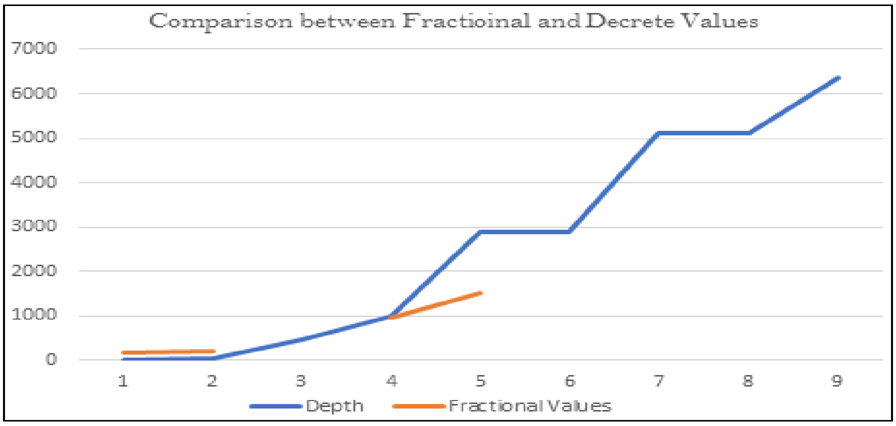
$$\alpha = \frac{1}{2} \quad into \; q = \lambda \frac{d}{dp}^{\alpha} (gradT) \qquad \text{such that } 0 < \alpha \leq 1$$

For $\alpha = \frac{1}{2}$

Table 5 Values of Temperature for $\alpha = \frac{1}{2}$ and Varying Depths

Layer of earth	Depth	Fractional Values
Crust	0	151.57
Mantle	33	211.5
	470	
	1000	950.02
	2900	1500.01
Outer core	2900	
	5100	2500.003
Inner core	5100	
	6371	3500.005

Figure 6 Comparison between Fractional and Discrete Values



Calculation of Pressure

Now we calculate the values of density at different layers of Earths by varying depths, it can be given by substituting

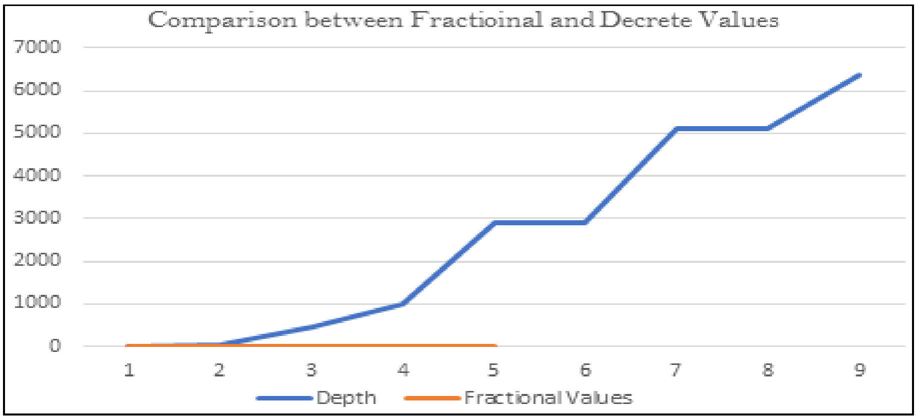
$$\alpha = \frac{1}{2} \; into \; P = \int_R^r g \frac{d}{dm}^{\alpha} (v) \, dr \qquad \text{such that } 0 < \alpha \leq 1$$

For $\alpha = \frac{1}{2}$

Table 6 Values of Pressure for $\alpha = \frac{1}{2}$ and Varying Depths

Layer of earth	Depth	Fractional Values
Crust	0	0.000005
Mantle	33	0.0017
	470	0.37
	1000	0.56
	2900	0.86
Outer core	2900	
	5100	1.56
Inner core	5100	
	6371	2.825

Figure 7 Comparison between Fractional and Discrete Values



RESULTS AND DISCUSSION

In the most devastating natural calamities, Earthquake is one of these, which results in high vibration of earth surface. It damages the infrastructure, roads and many other important things and not only this but it also significantly affects the surrounding environment and human's lifestyle. An earthquake can trigger many sudden changes in the environment which can be classified as primary (e.g. subsidence, surface faulting) and secondary effects (displaced rocks, tsunami, ground cracks, liquefactions, landslides). These effects are known as earthquake environmental effects (EEE). The scales which use these effects of an earthquake as a parameter for assessment are known as Earthquake Intensity Scales. These scales quantify an earthquake based on the observed effects on the surroundings. In this study fractional calculations are done and compared with discrete results.

In this research the study of the factors of earthquake is done because these factors play vital role in the change of the layers. This study conclude that layers of Earth depends on many factors like, pressure, density, temperature and gravitational acceleration. Consequently, all these factors directly don't depend upon time but pressure increase within the layers of Earth due to change in their mass with the passage of time which results in the change of density. So that temperature and gravitational acceleration changes as they directly depend upon density. So, all these factors change in term of time hence this can be discussed in Fractional Calculus.

CONCLUSION

Current study evaluates that factors which plays important role in the generation of Earthquake are first time discussed in term of Fractional Calculus and some new formulas of pressure, density, temperature and gravitational acceleration are proposed. Also, in this research proposed rules for α =1 and discrete value are verified which have relatively very small errors which proves the convergence of proposed results also some new values are calculated for temperature, density and gravitational acceleration for $\alpha = \frac{1}{2}$. In future this study can be extended with same factors and different time interval. That will be helpful to predict major change in the layers of Earth. As result Earthquake can be predictable.

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