THE STUDY OF NON-LINEAR BEHAVIOR OF KARACHI STOCK EXCHANGE DATA USING FRACTAL DIMENSION

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KEYWORDS	ABSTRACT
Karachi Stock Exchange, Fractal Dimension, Hurst Exponent and Linear Model	The research work done in this manuscript discusses the non-linear behavior of the Karachi Stock Exchange monthly index data (2007-2017). For this purpose, fractal dimension technique is used. The fractal dimension calculated for each year from 2007 to 2017. Fractal dimension indicates the persistency (1-1.5) behavior for each epoch. In high index data, year 2016 found more smooth (1.3474). Similarly, for low index data it is found that the year 2013 least rough (1.2951) among the all years. The study may be beneficial to understand the dynamics of Karachi Stock Exchange data. The results also show that the Karachi Stock Exchange monthly high and low index data can be forecast, as data sets are found persistent. On the basis of the results obtained we can say that the non-linearity of Karachi Stock Exchange as well as inflation rate may increase in future.

INTRODUCTION

We are living in an economic era and there is a number of challenges correlate with the stock exchange. During different political regimes in Pakistan play an important role to grow up and downfall for stock exchange. Karachi is the mega city and economic hub, mostly import and export carried out by Karachi sea port. The Karachi stock exchange is backbone for social/economic growth of Pakistan. The efficient market theory represents the price of financial assets traded on markets fully aware about available information (Gugten, 2010). The Efficient Market Hypothesis (EMH) theory was introduced by Fama which represents that financial markets are exceptionally capable of reflecting complete stock price information. The EMH is connected with the knowledge of a "random walk," which is a term casually entertained in the literature to illustrate series of prices where all succeeding changes in stock prices represent arbitrary differences from previous stock prices (Malkiel, 2003). The week day effect in study of KSE with the lowest stock returns on the first trading day like Saturday is reported by (Hussain 2000). Similarly, the daily returns for KSE and uncovered significant Tuesday effect reported by (Hussain, Hamid Akash & Khan, 2011).

This study tolerates the eleven years, KSE outflow behavior and it is favorable for stock investors. The study was started for the purpose of examining the existence of month effect in the KSE. In 2002, the KSE performance was awarded the best in emerging stock market by the Business week of the world. The KSE was established in 18th September, 1947 with total capital of 37 million rupees of five companies. The first index was KSE 50 index of fifty companies. In 1st November, 1991 KSE 100 index was first time introduced (Sohail & Javid, 2014). The cyclic pattern for KSE was observed in last seven decades. Normally the seasonal pattern is depicting calendar effects and indicates opportunities to analyze by stock and finance experts and social scientist (Bespalko, 2009). One of the most interesting topics in finance is the seasonality or calendar anomaly which has been widely studied in last three decades. Effect of Ramzan, week, day of week, month and January are some examples of stock price irregularities. Day and week effect is studied in

different part of globe and it is average return of stocks distributed uneven diverse day and week (Haroon & Shah, 2013).

MATERIALS AND METHODS

The research comprises the Karachi Stock Exchange (KSE) time series monthly data from 2007 to 2017. The fractal dimension was used to the stock exchange (High and Low of 100 indexes) data to know the performance of the trade in KSE. The linear model on fractal dimension was incorporated to observe the increasing or decreasing in the non-linearity of KSE.

Fractal Dimension

The fractal dimension is a technique that shows the smoothness (or roughness) of time series data. The expression (1) represents the well-defined relation of fractal dimension and Hurst exponent.

$$F + H = 2 \qquad (1)$$

The association of F and H in the above expression indicates that higher the value of F lower will be the H and vice versa. Usually self-similarity is measured using some power law, the power demonstrates scaling or similarity dimension (Falconer, 2007). The F show the self-similar object through self-similar pieces **r** scaled down using a factor **s** expresses as.

$$D = \frac{\ln r}{\ln s} \qquad (2)$$

The fractal dimension shows comparison of different numbers and shapes in perception of roughness and smoothness which are linked with fractals (Hassan, Abbas & Ansari, 2017). The fractal dimension's importance is because they are defined in construction with real world data, is measured approximately using experiments (Falconer, 2007). The clothes, clouds, feathers, coastlines, trees, neurons networks in body, distribution of frequencies, the colors emitted by sun are involved with the fractal dimensions (Michael, 1988). Consider the following scale of fractal dimension and Hurst exponent along with their relation.

Table 1 The Association of F and H

H	F	Correlation	Nature of Process
>0.5	<1.5	Positive	Persistent
=0.5	=1.5	Zero	Brownian
<0.5	>1.5	Negative	Anti-Persistent

The association of *F* and *H* is lasting by representing the correlation and nature of the time series data (Michael F. B, 1988; Hassan et al., 2017).

Linear Trend Model of Fractal Dimension

The linear trend models were also calculated for the KSE (low & high) time series data. The model shows the increasing trend for both the data sets.

$$y_{Fh} = 0.018\,x - 35.081 \qquad (3)$$

In (3), the model represents the increasing trend for fractal dimension of high index (Fh). The coefficient of determination (R^2) for the above model is found 0.5037. The range of R^2 is from 0 to 1.

$$y_{Fl} = 0.0135\,x - 25.969 \qquad (4)$$

Similarly, (4) indicates the linear trend model for the fractal dimension of low index of KSE. It is observed that the value of R^2 for model in (4) appear to be 0.2681 less than the R^2 of model in (3), depicts in the Figure 1.

RESULTS AND DISCUSSION

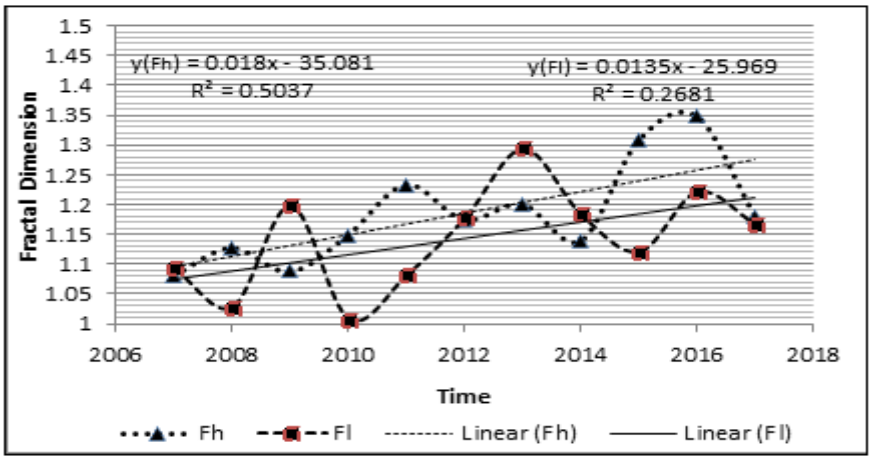
The city Karachi is the backbone and revenue generator of Pakistan and hence it highly depends on the trades and can be monitor by KSE. The study of KSE is much important for future planning and understanding the factors that are affecting the trades in Karachi. For this purpose, fractal dimension technique may be beneficial to understand the high and low fluctuation in the KSE data. The research was started by using monthly data but will be spread towards the daily and hourly data to understand the KSE precisely in next communication. The fractal dimension is calculated for each year starting from 2007 to 2017. Results indicates the persistency and self-similarity for each year, it means data can be forecast using suitable modeling. The year 2016 found smoothest for the last decade, depicts in table 1.

Table 1 Fractal Dimension and Hurst Exponent of High (*Fh*, *Hh*) and Low (*Fl*, *Hl*) KSE Index Monthly Time Series Data (2007-2017)

Year	Fh	Hh	Fl	Hl
2007	1.0812	0.9188	1.0926	0.9074
2008	1.1277	0.8723	1.0277	0.9723
2009	1.0877	0.9123	1.1989	0.8011
2010	1.1489	0.8511	1.0073	0.9927
2011	1.2335	0.7665	1.0818	0.9182
2012	1.1747	0.8253	1.1779	0.8221
2013	1.2005	0.7995	1.2951	0.7049
2014	1.1391	0.8609	1.1836	0.8164
2015	1.3072	0.6928	1.121	0.879
2016	1.3474	0.6526	1.2226	0.7774
2017	1.1808	0.8192	1.1667	0.8333

Results indicate that the year 2016 was the ideal in the perspective of the trade in Karachi, whereas year 2007 found least smooth (depicts in Figure 1).

Figure 1 The fluctuation of high and low KSE time series data in the perspective of fractal dimension, showing the close behavior at beginning and end of the selected data. Increasing of linear models depicts the increasing of non-linearity in KSE.



In 2007 there may be more fluctuation in the Karachi Stock market as compare to other chosen sets. The Karachi Stock Exchange market is highly correlated with the conditions (strike, lawlessness, political influences,) within the city. All selected years found positive

correlation and follows the fractional Brownian motion (fBm). Hurst exponent is also calculated to understand the long term behavior of Karachi stock exchange. The linear trend model of both (high & low) Karachi stock exchange index time series data shows the increasing in fractal dimension. Figure 1 depicts that the low index was less increase than the high index. It was also observed that coefficient of determination (R^2) for high index Karachi stock exchange trend found more than 0.5 (< 0.5037) means model is significance. It is also expected to continue increase trend in future. Increasing of the fractal dimension represents the increase in roughness of Karachi stock exchange 100 index. When fractal dimension of Karachi stock exchange (high index) reached to 1.5 then the behavior of Karachi stock exchange will represent the Brownian and difficult to forecast. In future, if fractal dimension crosses the 1.5 then behavior of Karachi stock exchange will be anti-persistent. The increasing trend of fractal dimension indicates the instability of trade in the city and in the country as well. The study may be useful to understand the behavior of Karachi stock exchange, so that government could take some action to control the instability in trade in Pakistan. Inflation rate may also enhance by increasing the fractal dimension.

CONCLUSION

The study may useful to understand the fluctuation in Karachi stock exchange monthly data. The Karachi stock exchange monthly time series is forecastable and can be simulate smoothly. Increasing of fractal dimension represents the increase in roughness of KSE 100 index. When fractal dimension of Karachi stock exchange (high index) reached to 1.5 then the behavior of Karachi stock exchange will represent the Brownian and difficult to forecast. On the basis of results, obtained in this research work, future planning along with the preventative from the affecting factors may also be design. This results of study are expected to provide significant information to the shareholders to have sufficient know-how about the non-linear behavior of the Karachi stock exchange. In this regard, the study might be beneficial to understand the dynamics and variation of Karachi stock exchange data.

REFERENCES

Bespalko, M. (2009). Calendar effects in daily bond returns: case of selected emerging economies (Doctoral dissertation, Kyiv School of Economics).

Falconer, K. (2007). Fractal geometry mathematical foundations and applications. Wiley & Sons. (2007).

Gugten, V. D. (2010). The Stock market calendar anomalies and macroeconomic news announcements. Erasmus University.

Haroon, M. A., & Shah, N. (2013). Investigating day-of-the-week effect in stock returns: Evidence from Karachi Stock Exchange - Pakistan. *Pakistan Journal of Commerce and Social Sciences*, 7(2), 381-393.

Hassan, D., Abbas, S., & Ansari, M. R. K. (2017). Comparative study of solar flare north-south hemispheric and K-index geomagnetic activity time series data using the fractal dimension. *Gomal University Journal of Research*, *33*(1), 72-79.

Hussain, F. (2000). The day of the week effect in the Pakistani equity market: An investigation. *The Lahore Journal*, 5(1), 93-98.

Hussain, F., Hamid, K., Akash, R. S., & Khan, M. I. (2011). Day of the week effect and stock returns: Evidence from Karachi Stock Exchange, Pakistan. *Far East Journal of Psychology and Business*, 3(1), 25-31.

Malkiel, B. G. (2003). The efficient market hypothesis and its critics. *The Journal of Economic Perspectives*, 17(1), 59-82.

Michael F. (1998). Barnsley: Fractals everywhere. *Academic Press*, ISBN 978-0-12-079062-3, pp. I-XII, 1-394.

Sohail, A., & Javid, A. Y. (2014). The Global Financial Crisis and Investors' Behaviour: Evidence from the Karachi Stock Exchange (No. 2014: 106). Pakistan Institute of Development Economics.