

## **CONCRETE OPERATIONAL STAGE OF PIAGET'S COGNITIVE DEVELOPMENT THEORY: AN IMPLICATION IN LEARNING MATHEMATICS**

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### **ABSTRACT**

*This research study was conducted with the aim to apply Concrete Operational Stage (7-11 years) of Piaget's Cognitive development theory in learning mathematics. Objectives of the study were; to apply Concrete Operational Stage (7-11 years) of Piaget's Cognitive development theory on students of age seven to eleven years at district Bannu, to compare academic achievement of urban & rural students in the subject of mathematics after application of Concrete Operational Stage (7-11 years) of Piaget's Cognitive development theory. This study was survey type in nature. Population of the study were all male students of age seven to eleven years studying in public sectors schools of district Bannu. Sample of the study were comprised of two hundred students of age seven to eleven years. Mean, standard deviation and t-test was used as statistical tests for analysis of data. Results of the study revealed that students can do conservation of numbers, ordering, and reversibility to some extent in our urban & rural context while students can do logic successfully.*

**Keywords:** *Piaget's Theory, Cognitive development, Concrete operational stage.*

### **INTRODUCTION**

Jean Piaget was born in 1896 at Switzerland. After completion of his doctoral degree at the age of 22 years; Piaget formally started work on both psychology and education. After taking valuable guidance from Alfred Binet, Piaget took an interest in the cognitive development of children. After his long observations, he concluded that children were not less intelligent than adults, they simply think differently. The stage theory of Piaget explains the cognitive development of children. Piaget's theory of cognitive development is a comprehensive theory first developed by Jean Piaget. It is primarily known as a developmental stage theory. It deals with the nature of knowledge itself that how humans come gradually to acquire, construct, and use it. To Piaget, cognitive development was a progressive reorganization of mental processes as a result of biological maturation and environmental experiences.

Piaget's theory on cognitive development consists of four stages: Sensorimotor: (birth to 2 years), Preoperations: (2 to 7 years), Concrete operations: (7 to 11 years), and Formal Operations: (12 to 16 years). One contribution of Piagetian theory concerns the developmental stages of children's cognition. His work on children's quantitative development has provided mathematics educators with crucial insights into how children learn mathematical concepts (Ojose, 2008). Mathematics is considered as a dry subject at school level and hence students' low achievement in mathematics has always been under discussion among teachers, parents and researchers (Vygotsky, 1986). Mathematics is an important part of cognition and thinking. Issues concerning mathematical thinking are one of the fundamental goals of mathematics curricula and instruction (Lutfiya, 1998).

Mathematics is a core subject in school curriculum of Pakistan up to Secondary Education. The curriculum is organized around contents strands of number and number operations, measurement, geometry, data handling and algebra. It has explicitly stressed in the objectives to enable students to think logically, reason systematically and make discerning conjectures. Therefore, this study was aimed to apply the concrete operations stage (7-11 years) of Piaget's Cognitive development theory on students of age seven to eleven years at district Bannu in learning mathematics concepts particularly conservation of numbers, ordering, reversibility and logic in rural and urban context.

### **Statement of the Problem**

The statement of the problem was to apply the concrete operations stage (7-11 years) of Piaget's Cognitive development theory on students of age seven to eleven years in learning mathematics concepts in rural and urban context at district Bannu.

### **Objectives of the Study**

The following were objectives of the study

1. To apply Concrete Operational Stage (7-11 years) of Piaget's Cognitive development theory in learning mathematics concepts (viz; conservation of numbers, ordering, reversibility and logic) on students of age seven to eleven years at district Bannu.
2. To compare academic achievement in mathematics concepts (viz; conservation of numbers, ordering, reversibility and logic) of rural and

urban students falling in Piaget's Concrete Operational Stage (7-11 years) of cognitive development theory.

### **Hypotheses of the study**

This study was guided by the following null hypotheses

1. There is no significant difference in the concrete operational cognitive development stage of rural and urban students for the concept of conservation of numbers in learning Mathematics.
2. There is no significant difference in the concrete operational cognitive development stage of rural and urban students for the concept of ordering in learning Mathematics.
3. There is no significant difference in the concrete operational cognitive development stage of rural and urban students for the concept of reversibility in learning Mathematics.
4. There is no significant difference in the concrete operational cognitive development stage of rural and urban students for the concept of logic in learning Mathematics.

### **Significance of the Study**

The study was significant in the following perspective

This study may show reliable results about Piaget's theory on cognitive development in concrete operational stage students (7-11 years) in learning mathematical concepts particularly conservation of numbers, ordering, reversibility and logic in rural and urban context. It will be helpful for Policy makers, curriculum developers to suggest appropriate concepts keeping in view of Piaget's concrete operational stage. This study would also be helpful for those teachers who are teaching mathematics at primary and elementary level.

### **LITERATURE REVIEW**

There's a big difference between Cognition and Intelligence. Cognition is about the way your brain thinks, and includes awareness, perception, reasoning, and judgment. Intelligence is the ability to understand, learn and profit from experience (Vygotsky, 1981). Science affirmed that cognitive development is the improvement of thinking processes through which knowledge is obtained. Cognitive development count awareness, insight, instinct, way of thinking, analysis, logic and reckoning.

Piaget, (1977) explained that the sensorimotor stage starts from zero to two years. It is first stage in cognitive development of children. In the sensorimotor stage, awareness of the environment is creating in infants by coordinating practices through seeing something and hearing some sound along with physical action e.g. shaking hands. Infants achieve information, awareness, understanding, facts, and perception of the surrounding environment through physical and motor actions. He or she also gains knowledge through reflex and automatic action throughout the sensorimotor stage.

The 2<sup>nd</sup> stage of Piaget's theory on cognitive development is known as preoperational stage and it starts from two years age of the children and ends with the closing of seven years age. In the beginning, children develop their language. During this age, children are able to develop symbolic play and manipulate symbols, seeing order of games. Piaget observed that they have to face problems in seeing everything with various outlooks. Moreover, they did not know concrete things, logic and cannot work their thoughtful information. Piaget stated that the end of 2<sup>nd</sup> year age of children is called a new type of psychological performance.

The 3<sup>rd</sup> stage of Piaget's cognitive developmental theory is called concrete operational stage. After the preoperational stage, the concrete operational stage begins with the children age of seven years and ends with the age of eleven years. Its characteristics are proper exercise of logic, reasoning, resolve difficulties, problems, and go through hindrances in a logical way. A child's thinking power is turned into further developed as well as grown up like adulthood in this stage. Children can solve those problems which are relevant to concrete actions or things. However, conceptual, theoretical and speculative thoughts have not developed in children so far. Copeland (1979) stated that this stage marks the beginning of logico-mathematical thought. The child is said to be "operational" in his thinking.

The stage is called concrete operational. The child no longer uses perception or sensory cue as a basis for answering questions requiring logical thought. Eggen and Kauchak, (2000) explained that in concrete operational periods discuss classification; seriation; number concepts, temporal concepts, spatial concepts and acquisition of information. Adult often thinks of mathematics as an abstract discipline involving complex algebraic formulas and geometric calculations. Yet, the foundations of math are grounded in concrete experience. The cognitive

concepts such as classification, seriation, numbers, time, and spatial concepts are all an integral part of the development of mathematical knowledge.

The last stage is known as formal operational stage (12 to 16 years). Intelligence or cleverness is confirmed owing to the logical or reasonable ways and using symbols connected with theoretical concepts. In formal operations stage, human being has ability of theoretical plus deductive logic or way of thinking. Piaget thought that deductive logical way of thinking is significant throughout this period. Abstract thought also emerges at this period. Eggen and Kauchak, (2000) stated that the children formal operations stage is characterized by complicated, theoretical or conceptual thoughts plus rational way of thinking capabilities practical to physical, societal in addition to ethical troubles.

Piaget's theory on cognitive development presents a complex picture and Piaget did not suggest specific educational applications of his work, but educators have transformed his theory into actual models more than any other (Eggen & Kauchak, 2000). Jean Piaget's theory on cognitive development has been condemned on the basis or reasons as to theoretically incomplete, practically untrue, or thoughtfully and epistemologically unsustainable. Piaget replied to the condemnation by recognizing that they did not know the results as he desired to acquire commencing his study (Zimmerman & Whitehurst, 1979). Piaget did not explicitly relate his theory to education, although later researchers have explained how features of Piaget's theory can be applied to teaching and learning. According to Piaget's theory children should not be taught certain concepts until they have reached the appropriate stage cognitive development. As several studies have shown Piaget underestimated abilities of children and his tests were sometimes confusing or difficult to understand (Hughes, 1975).

Vygotsky (1978; 1981; 1986) criticized Piaget's theory on cognitive development first. Vygotsky's structure propose that societies culture or way of life are dissimilar however changing, in the direction of a general (socio-historical) objective as well as culturally dissimilar type of presentation on everyday jobs could be dealt through suitable learning involvement (Matusov, 2007; Craig and Miller, 1984; Moll, 2004). Eggen & Kauchak, (2000) stated that opponent of Piaget's research effort claim that Piaget's suggested theory does not present entire report of cognitive development. Zimmerman & Whitehurst, (1979) stated that though it is not feasible to educate cognitive progress openly, study has established that cognitive development be able to accelerate. However, Piaget's

theory has produced a lot of research or study which has improved our awareness or knowledge of cognitive development. Keating (1979) stated that Piaget was not succeeded to think about the impact of societal surroundings plus culture might influence on cognitive development (re: Vygotsky).

## RESEARCH METHODOLOGY

### Research Design

This study was survey type in nature.

### Population

Population of the study was all male students of age seven to eleven years studying in public sectors schools of district Bannu.

### Sample

Sample of the study was comprised of two hundred students of age seven to eleven years who were selected randomly.

### Instrument

A self-developed objective type test plus concrete operational activities of mathematics was constructed and used to collect data from the sample. A self-developed objective type test plus concrete activities in learning mathematics concepts particularly conservation of numbers, ordering, reversibility and logic was administered on students of age seven to eleven years at district Bannu. Two hundred male students (one hundred from urban and one hundred from rural students) were selected from public sector schools of district Bannu using random methods.

## FINDINGS OF THE STUDY

The collected data was analyzed through SPSS 16 using mean, standard deviation and t- test. The following cut points of the mean values were used to interpret the descriptive results:

Below average value = (0-0.67),  
 Average value = (0.68-1.33),  
 Above average value = (1.34-2).

**Table 1:** Showing scores, %age, Mean and SD on Conservation of Numbers.

Q N	Questions	f	Scores	%age	M	SD
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1	Some milk in one cup will be same when I pour it into two cups.	125	0	62.5	0.75	0.97
		75	2	37.5		
2	A child is shown two different glasses containing the same amount of water. The water in one glass is then poured into a taller glass with smaller diameter. Tell me, the amount of water is same or not.	124	0	62	0.76	0.97
		76	2	38		
Overall scores		249	0	62.25	0.76	0.73
		151	2	37.75		

Results show the academic achievement of the students regarding the ability of conservation of numbers falls in the range (0.68-1.33) of average value with  $M = 0.76$  and  $SD = 0.73$ .

**Table 2:** Showing scores, %age, Mean and SD on Ordering

Q N	Questions	F	Scores	%age	M	SD						
3	Complete the blocks in order.  <table border="1" style="margin-left: 40px;"> <tr> <td style="width: 20px; height: 20px;">1</td> <td style="width: 20px; height: 20px;">2</td> <td style="width: 20px; height: 20px;">3</td> <td style="width: 20px; height: 20px;"></td> <td style="width: 20px; height: 20px;"></td> <td style="width: 20px; height: 20px;">6</td> </tr> </table>	1	2	3			6	5	0	2.5	1.95	0.31
		1	2	3			6					
195	2	97.5										
4	Indicate number from one to six on a given arrow 	64	0	32	1.36	0.94						
		136	2	68								
5	Order the following fractions from largest to smallest. 1/10, 2/8, 3/4, 1/15, 7/8 and 5/12.	188	0	94	0.12	0.48						
		12	2	6								
6	Ten dolls of different sizes in one row and ten balls of different sizes in another row. Match them by size.	164	0	82	0.36	0.77						
		36	2	18								
Overall scores		421	0	52.63	0.95	0.3						
		379	2	47.37								

Results show the academic achievement of the students regarding the ability of ordering falls in the range (0.68-1.33) of average value with  $M = 0.95$  and  $SD = 0.3$ .

**Table 3:** Showing scores, %age, Mean and SD on Reversibility

Q N	Questions	F	Scores	%age	M	SD
7	+A-A=0 or +1-1=0      True/ False	145	0	22.5	1.55	0.84
		155	2	77.5		
8	If A=B then B≠A      True/ False	104	0	52	0.96	1.00
		96	2	48		
Overall scores		149	0	37.25	1.26	0.69
		251	2	62.75		

Results show the academic achievement of the students regarding the ability of reversibility falls in the range (0.68-1.33) of average value with  $M= 1.26$  and  $SD= 0.69$ .

**Table 4:** Showing scores, %age, Mean and SD on Logic

Q N	Questions	F	Scores	%age	M	SD
9	If $A > B$ and $B > C$ then $A > C$ True/ False	89	0	44.5	1.11	1.00
		111	2	55.5		
10	How many brothers do you have?	18	0	9	1.82	0.57
		182	2	91		
11	How many sisters do you have?	19	0	9.5	1.81	0.58
		181	2	90.5		
12	How many brothers and sisters altogether?	27	0	13.5	1.73	0.69
		173	2	86.5		
Overall scores		153	0	19.13	1.62	0.50
		647	2	80.87		

While results show the academic achievement of the students regarding the ability of logic falls in the range (1.34-2) of above average with  $M= 1.62$  and  $S.D= 0.50$ .

**Table 5:** Comparison of rural & urban students Conservation of Numbers

Location	N	M	SD	t-value	P-value
Rural	100	0.91	0.67	_3.75	0.00*
Urban	100	1.18	0.74		

d.f. = 398

\* $p \leq 0.05$

The p- value (0.00\*) results show that there is a significant difference between the academic achievement of rural and urban students at 0.05 level of significance and therefore, there is no significant difference was rejected. However, it can be inferred that the performance regarding conservation of numbers of the urban students with mean score  $M= 1.18$  and  $SD=0.74$  was better than the rural students with mean score  $M= 0.91$  and  $SD= 0.67$ .

**Table 6:** Comparison of rural and urban students for the ability of Reversibility

Location	N	M	SD	t-value	P-value
Rural	100	1.22	0.64	1.36	0.18
Urban	100	1.23	0.76		

d.f. =398

\*  $p \leq 0.05$

The p- value (0.18) results show that there is no significant difference between the academic achievement on reversibility of rural and urban students at 0.05 level of significance and therefore, there is no significant difference was accepted. However, It can be inferred that the performance regarding reversibility of the urban students with mean score  $M= 1.23$  and  $SD= 0.76$  was better than the rural students with mean score  $M= 1.22$  and  $SD= 0.64$ .

**Table 7:** Comparison of rural and urban students for the ability of Ordering

Location	N	M	SD	t-value	P-value
Rural	100	.84	0.44	_3.466	0.00*
Urban	100	0.98	0.39		

d.f. =398

\*  $p \leq 0.05$

The p- value (0.00\*) results show that there is a significant difference between the academic achievement on ordering of rural and urban students at 0.05 level of significance and therefore, there is no significant difference was rejected. However, it can be inferred that the performance regarding ordering of the urban students with mean score  $M= 0.98$  and  $SD= 0.39$  was better as compared to the rural students with mean score  $M= 0.84$  and  $SD= 0.44$ .

**Table 8:** Comparison of rural and urban students for the ability of Logic

Location	N	M	SD	t-value	P-value
Rural	100	0.92	0.79	_3.98	0.00*
Urban	100	1.22	0.71		

d.f. =398

$p^* =0.05$

The p- value (0.00\*) results show that there is a significant difference between the academic achievement on logic of rural and urban students at 0.05 level of significance and therefore, there is no significant difference was rejected. However, it can be inferred that the performance regarding logic of the urban students with mean score  $M= 1.22$  and  $SD= 0.71$  was better than the rural students with mean score  $M= 0.92$  and  $SD= 0.79$ .

## DISCUSSION

The academic achievement of the students falling in Piaget's concrete operational stage (7-11 years) regarding the ability of logic indicates the students overall mean scores (1.62) falls in the range (1.34-2) of above average with  $M= 1.62$  and  $S.D= 0.50$  and students can do logic ability in learning mathematics. While the

academic achievement of the students falling in Piaget's theory on concrete operational stage (7-11 years) regarding the ability of conservation of numbers, ordering, reversibility in learning mathematics indicates the students overall mean scores (0.76), (0.95),(1.26) falls in the range (0.68-1.33) of average value with  $M= 0.76,(0.95),(1.26)$  and  $SD = 0.73,0.3, 0.69$  respectively.

It means that students can do conservation of numbers, ordering, and reversibility to some extent in learning mathematics in our rural and urban context. According to Piaget (1964), these operations at the concrete level include those of classification, ordering, conservation of number, sequential operations, and simple logic of classes. Keeping in view of Piaget's Theory on concrete operations stage (7-11 years), this study revealed partially the concrete operations stage (7-11 years) of Piaget's Cognitive development theory on students of age seven to eleven years in our rural and urban context at district Bannu in learning mathematics concepts especially conservation of numbers, ordering, and reversibility.

In table # 6, the p- value (0.18) reflects that there is no significant difference between the academic achievement on reversibility of rural and urban students falling in Piaget's concrete operational stage (7-11 years) at 0.05 level of significance and the null hypothesis was accepted. While the p- values (0.00\*) shows that there is no significant difference between the academic achievement on conservation of numbers, ordering, logic in mathematics in rural and urban students at 0.05 level of significance and therefore, the null hypothesis were rejected. However, it can be inferred that the performance regarding conservation of numbers, ordering, reversibility and logic in mathematics of the urban students with mean score  $M= 1.18, 0.98, 1.22, 1.23$  and  $SD=0.74, 0.39, 0.71, 0.76$  were better than the rural students with mean score  $M= 0.91, 0.84, 0.92, 1.22$  and  $SD= 0.67, 0.44, 0.79, 0.64$ .

Piaget (1964) stated that Piaget theory on cognitive development have been challenged on many reasons with various angles. Vygotsky supported significance of a child's cultural environment deep effects on development periods. As various cultures or societies have dissimilar societal connections; these ideas challenged Piaget's theory on cognitive development. Seeing Vygotsky views, this study revealed that socio-cultural differences have impact on the concrete operations stage (7-11 years) of Piaget's Cognitive development theory on students of age

seven to eleven years in rural and urban context at district Bannu in learning mathematics concepts because urban students were better than the rural students.

## **CONCLUSIONS**

It was concluded on the basis of data analysis and results after applications of Piaget's theory on concrete operational stage (7-11 years) that students of age seven to eleven years can do conservation of numbers, ordering, reversibility to some extent and logic fully in our rural and urban context. Urban school students falling in Piaget's concrete operational stage (7-11 years) have better performance regarding the ability of conservation of numbers, ordering, reversibility and logic in mathematics than the rural students. It was concluded that socio-cultural differences have impact on students of the concrete operations stage (7-11 years) of Piaget's Cognitive development theory. However, there are also some weaknesses in our text books which need to refine for its improvement.

## **IMPLICATIONS AND RECOMMENDATIONS**

1. It is recommended that comprehensive learning activities should be started for students in learning mathematics at primary and elementary level.
2. Text books should be revised according to the mental level of students falling Piaget's concrete operational stage (7-11 years) students in learning mathematics.
3. Teaching strategies in learning mathematics should be revised at concrete stage.
4. Government should allocate special incentive fund yearly e.g. scholarships, computer labs, facilities, etc for enhancing cognitive interest of the students.
5. It is recommended that inter-schools students visits, competition, experimental learning activities should be started to motivate cognitive activities of the rural school students.
6. Policy makers and curriculum developers are suggested to design activity based curriculum in learning mathematics at primary and elementary level.
7. The same study may also be conducted at national level to achieve reliable results.

## **REFERENCES**

Copeland, F., & Richard, W. (1979). *How Children Learn Mathematics; Teaching Implications of Piaget's Research*, 3rd Edition, Macmillan Publishing Co., Inc.

- Craig, A., & Miller, R. (1984). Mother-Child Interaction. In, Edwards S.D. (Ed.), *Development and Enrichment Strategies for Disadvantaged Children*. Kwadlangezwa: University of Zululand.
- Engen, P. D., & Kauchak, D. P. (2000). *Educational psychology: Windows on classrooms* (5th ed.). Upper Saddle River, NJ: Prentice Hall.
- Hughes, M. (1975). *Egocentrism in preschool children*. Unpublished doctoral dissertation. Edinburgh University.
- Keating, D. (1979). Adolescent thinking. In J. Adelson (Ed.), *Handbook of adolescent psychology*, 211-246. New York: Wiley.
- Lutfi, A.L. (1998). Mathematical thinking of high school students in Nebraska. *International Journal of Mathematical Education in Science and Technology*. 29(1), 55-60.
- Matusov, E. (2007). Applying a sociocultural approach to Vygotskian academia: Our star is not like yours, and yours isn't like ours. *Culture and Psychology*.
- Mayesky, Mary. (2001). *Creative activities for young children*, 7th edition Delmar Cengage Learning Publisher.
- Ojose, Bobby. (2008) Applying Piaget's Theory of Cognitive Development to Mathematics Instruction. Retrieved on July 05, 2013, from: [http://math.coe.uga.edu/tme/Issues/v18n1/v18n1\\_Ojose.pdf](http://math.coe.uga.edu/tme/Issues/v18n1/v18n1_Ojose.pdf).
- Piaget J. (1964). Development and Learning, *Journal of Research in Science Teaching*, 3.
- Piaget, J. (1977). *Epistemology and psychology of functions*. Dordrecht, Netherlands: D. Reidel Publishing Company.
- Vygotsky, L. S. (1981). The genesis of higher mental functions. In Wertsch, J.V. (Ed.). *The Concept of Activity in Soviet Psychology*. New York: Sharpe.
- Vygotsky, L. S. (1978). The prehistory of written language. In M. Cole, V. John-Steiner, S. Scribner, & E. Souberman (Eds.), *Mind and Society: The development of higher psychological process* (105-119).
- Vygotsky, L. S. (1986). *Thought and language* (A. Kozulin, Trans.). Cambridge, MA: MIT Press. (Originally published 1934).
- Zimmerman, G. J., & Whitehurst, B. J.,(1979). *Structure and function: A comparison of two views of the development of language and cognition*, 1-22. New York: Academic Press.