The Gomal University Journal of Research
Vol. 2 No. 1 (September 1982) pp. 119 – 122

ON THE EQUATIONS OF MOTION BND THE QUANTUM MECHANICAL COMMUTATION RELATIONS

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ABSTRACT

The Commutator of the Hamiltonian with the operator corresponding to a physical quantity gives the operator which corresponds to the time derivative of the quantity. We apply this principle to the Hamiltonian for a free particle and harmonic oscillator. We also attempt to investigate the postulate, that the quantum mechanical operators obey the classical equations of motion and that they uniquely determine the commutation relations.

INTRODUCTION

Schrodinger obtained his wave mechanical equation by assuming that the wave motion correspond to the classical motion of a particle if the force of field in which the wave is moving changes slowly with position. Assuming that the Hamiltonian has the simple form $H = \frac{p^2}{2m} + V(x)$ ...(1)

The above statement can be summarised by observing that the operators in the Heisenberg relation satisfy the classical differential equations :

$$q' = \frac{P}{m} \; ; \quad p' = - \frac{\partial V}{\partial x} \qquad ...(2)$$

Since the time derivative of any operator is its commutator with the Hamiltonian, so that (1) is equivalent to (2) and thus

$$\left(\frac{i}{\hbar}\right)[H, q] = \frac{p}{m} \; ; \; \left(\frac{i}{\hbar}\right)[H, p] = - \frac{\partial V}{\partial x} \qquad ...(3)$$

or

$$\left(\frac{i}{\hbar}\right)[\tfrac{1}{2} p^2, q] = p \; ; \; \left(\frac{i}{\hbar}\right)[V, p] = - \frac{\partial V}{\partial x} \qquad ...(4)$$

We know that Dirac's equation of the electron does not lead to the classical

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