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THE LAWS OF NILPOTENT-POINTED GROUPS

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1. INTRODUCTION

R.C. Lyndon [1] proved that every nilpotent group variety has a finite basis for its laws. (That is, there is a finite set of laws of which every law is a consequence). Here we shall examine the analogous statement for nilpotent pointed groups, where a *pointed group* is a pair (G, g) consisting of a group G togather with a distinguish element g of G.

By a law of a pointed group (G, g) we shall mean a word w of the free group on the countable set $\{y, x_1, x_2, ---\}$ such that w always becomes equal to the identity element of G when g is substituted for y and arbitrary elements of G are substituted for $x_1, x_2, ----$ (For example, $[y, x_1]$ is a law of (G, g) if g is central in G or G is abelian). Included among the laws of (G, g) are the laws of the group G, or more precisely, those words in $x_1, x_2, ----$, which are laws of G. Thus the idea of laws of a pointed group generalises the idea of laws of a group. Also, our pupose in this paper is to generalise the result proved by R.C. Lyndon in [1], that every variety of nilpotent pointed groups is finitely based. Detailed information concerning varieties of groups may be found in [2].

A pointed group may be regarded as a group with an extra nullary operation, so it is an algebra in the sense of universal algebra. The more general concepts relating to universal algebra and varieties of algebra are described in [3]. It is useful to note the form of some of these concepts take for pointed groups.

As indicated above, the factor algebra of (G, g) is (G/N, gN) where N is a normal subgroup of G. The subalgebra of (G, g) is (H, g) where H is a subgroup

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