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ON A CLASS OF TWO-GENERATORS, TOO-PELATIONS FINITE AND INFINITE GROUPS

by

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In [2] Johnson and Robertson has shown that the metacyclic group $< a, b \mid a^m = 1, b^{-1} ab = a^r , b^n = a^s >$ is defined by two generators and two relations in the case when $s = m/(m, r-1)$. In this note we give a proof for more or less similar problem of describing the structure of the group $< a\ b \mid a^{2m} = b^n , a^{-r}\ ba^r = b^{n-1} >$, clearly, for $r = 1$, this group is metacyclic of order 2 mn (n—2). However, we state and prove the following theorem.

Theorem :

Let $G = < a, b \mid a^{2m} = b^n , a^{-r}\ b\ a^r = b^{n-1} >$. Then if

(i) 2m, r) = 1, n > 2, G is metacyclic of order 2mn (n—2)

(ii) (2m, r) ≠ 1, 2m ≠ r, n ≧ 2, G is in-finite.

Proof :

(i) To prove the result, it is enough to show that $\lhd\ b >$ is a normal subgroup of G. To do this, since (2m, r) = 1, there exist integers α, β such that $2m\ \alpha\ \beta\gamma + = 1$

Now since $a^{-r}\ b\ a^r = b^{n-1}$, we have

$$a^{-\beta\gamma}\ b\ a^{\beta\gamma} = b^{(n-1)^{\beta}} \text{......(i)}$$

Since a^{2m} is central, so (i) gives

$$a^{-(2m\ \alpha+\beta\gamma)}\ \ b\ \ a^{(2m\ \alpha+\beta\gamma)} = b^{(n-1)^{\beta}}$$

$$=> a^{-1}\ \ b\ a = b^{(n-1)^{\beta}}$$

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